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Theory of vertical oscillations of frontally mounted top-gathering machine

V. Bulgakov, Dr. Sc., academician of NAAS,
National University of Bioresources and nature use of Ukraine
V. Adamchuk, Dr. Sc., academician of NAAS,
National Science Center "Institute of Mechanization and Electrification Agriculture"
Ye. Ihnatyev, eng.,
Tavria state agrotechnological university

The purpose. To determine influence of kinematic and design parameters of top-gathering machine upon quality of clear cut of haulm at oscillations in longitudinal-vertical plane of top-gathering apparatus at frontal mounting on wheel tilling and aggregating tractor. **Methods.** Methods of simulation which are based on principles of higher mathematics, theoretical mechanics, on the solution of system of differential equations, creation of programs and numerical solution on PC. **Results.** Equivalent scheme is developed for top-gathering machine, which is frontally mounted on wheel row-crop tractor at its oscillations in longitudinal-vertical plane, with consideration of all forces which originate in view of kinematic initiation which originates at driving on soil surface with irregularities in row widths of sowings of sugar beet, and also owing to elastic-and-damp properties of its support-patterning wheels. **Conclusions.** By results of numerical modeling final expression is gained. That allowed determining vertical movement of the lower end of rotor top-gathering apparatus at oscillations of the top-gathering machine in longitudinal-vertical plane.

Key words: *sugar beet, haulm, harvesting, rotor top-gathering apparatus, oscillations, differential equations, amplitude, frequency.*

Statement of a problem. High-performance and high-quality cleaning of a tops of vegetable of sugar beet remains rather complex and actual challenge of area of beet breeding. Recently in the world the multistage way of cleaning of a tops of vegetable at the heart of which the continuous main cut of all array of a tops of vegetable (on a harvester width of cut), its collecting and transportation in nearby the going vehicle is carried out in the beginning, and further, with use of individual copying of each head of a root crop in a row most was widely adopted, after-treatment or a cleaning (or at the same time is provided: both after-treatment and a cleaning different working bodies) heads of root crops from the remains of a tops of vegetable. As the specified operations are carried out consistently for the root crops of sugar beet which are in the soil (i.e. on a root) and cleaning of a tops of vegetable precedes operation a digging up of root crops of beet from the soil, beet tops harvesting units as independent farm vehicles, or beet tops harvesting modules as compound units of beet combines, surely are located in frontal situation in relation to power means (to a tractor — if it is about mounted beet tops harvesting machines or to front part of b of self-propelled beet combines). However, it is established by the pilot studies conducted by us that in the course of work, the beet tops harvesting unit which is frontally hung on a tractor carries out three-dimensional motions which are defined by a field surface relief, the forward speed of the movement of a tractor, placement of gage wheels concerning system of subweight of the car, etc. that significantly influences quality of performance of this technology process. Use as the copying — pneumatic wheels causes fluctuations of the beet tops harvesting unit in the vertical plane which will influence most quality of performance of technology process — uniform cutting of a tops of vegetable from heads of root crops on all width of cut, its fullest collecting and transportation without loss.

Analysis of the last researches and publications. Despite a wide circulation of frontally hung beet tops harvesting modules of beet pullers of the western production, and also some designs of the domestic frontally hung beet tops harvesting units, it did not cause analytical researches of their oscillating motion.

However, using the technique stated in [1] it is possible to construct settlement mathematical model of this car which will give the chance to study influence of its design data on the movement on ranks of root crops of sugar beet and to roughnesses of a surface of the soil.

Research objective. To define influence of kinematic and design data of the beet tops harvesting unit which is frontally hung on a wheel tractor, at a size of amplitude of fluctuations in the longitudinally vertical plane of the beet tops harvesting device.

Research methods. When performing this research methods of creation of settlement mathematical models of functioning of farm vehicles and machine units, based on theoretical mechanics, the higher mathematics, methods of drawing up programs and the numerical decision of systems of the differential equations on the personal computer are used.

Results of research and their analysis. We developed the new universal beet tops harvesting car which is carrying out technology process of cleaning of a tops of vegetable of sugar beet by the principle of the mower grinder which is frontally hung on wheel, row crop an integral tractor. In this car the rotational beet tops cutting device in which the cutting-off knives pivotally established on a drive drum have a bow-shaped form is used and, rotating in the longitudinally vertical plane, provide a unsupported, non-sensing cut of the main array of a tops of vegetable from all width of cut [9-11]. In fig. 1 the constructive and technology scheme of this machine and tractor unit which provides a cut as bunches and leaves of a tops of vegetable of sugar beet, and plants which are on a sugar-beet field at the time of cleaning is shown and transports all cut-off weight in a body of the vehicle which moves near it.

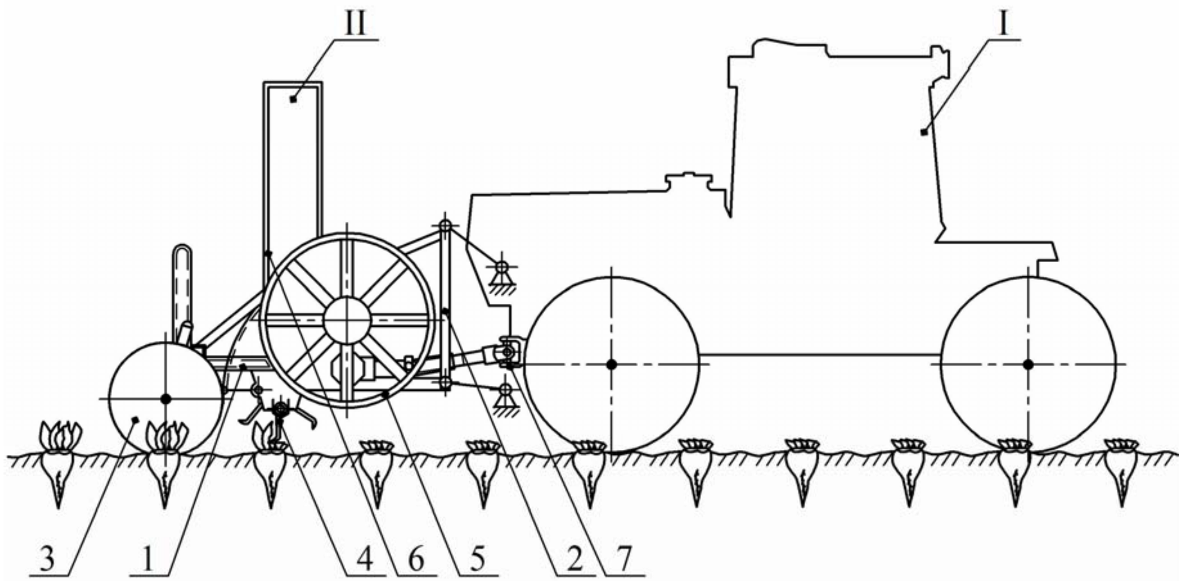

Fig. 1. Constructive and technology scheme of the machine and tractor the unit for cleaning of a tops of vegetable of sugar beet: I — wheel, row crop an integral tractor; II — frontally hung beet tops harvesting unit: 1 — frame; 2 — top strut assembly; 3 — the basic pneumatic gage wheel; 4 — rotor cutting device; 5 — the transporting working body; 6 — charging device; 7 — the unit drive from a front power take-off shaft of a tractor

Technology process of cleaning of a tops of vegetable of sugar beet happens as follows. At the movement of a row-crop wheel integral tractor (with narrow tires) on rows of crops of sugar beet, the pneumatic gage wheels 3 located in front part of the movable frame 1 install the rotor of beet tops cutting device 4 with knives on the necessary height of a cut. Knives have a bow-shaped form, and are pivotally established on cylindrical forming on length of a rotor 4 in such a way that provide overlapping of all width of cut. Knives rotate with a big frequency thanks to what the cut of all array of a tops of vegetable is provided. Absolute speed of the ends of edges of bow-shaped knives for a cut of a tops of vegetable reaches 20... 25 $m \cdot s^{-1}$, and for bevelling of others, in particular thick-stemmed cultures – 40... 50 m/s [11].

The tops of vegetable which is cut off by bow-shaped knives moves to upper part of a casing where gets on the screw conveyor which moves the cut-off weight to face part of the car then the transporting working body 5 via the charging device 6 unloads it in a body of the vehicle moving near the beet tops harvesting unit. The drive 7 of the beet tops harvesting unit is carried out from a front power take-off shaft of the aggregating row-crop tractor. Finally technology process of cleaning of a tops of vegetable comes at after-treatment of heads of root crops from the remains of a tops of vegetable the cleaner of heads established behind the aggregating row-crop integral tractor.

The main advantage of the beet tops harvesting unit of rotational type is that, having only one working body — a rotor with knives, it provides a qualitative cut of bulk of a tops of vegetable and its transportation in the vehicle moving nearby or can scatter the crushed tops of vegetable on the cleaned part of a beet field. The beet tops harvesting unit universal, has high reliability and can be used as a rotary mower (i.e. can qualitatively cut off different herbage up to 1 m high) [11]. It is necessary to refer excessive crushing of a tops of vegetable in case of its collecting in a vehicle body, some pollution of collected mass of a tops of vegetable to shortcomings of the beet tops harvesting unit of rotational type soil impurity, especially during the work on the dry soil and at installation of the cutting device on the lowered cut height, considerable difficulties of engineering service (in particular at removal of bow-shaped knives for sharpening, replacement and so forth).

For definition of influence of design and kinematic data of the beet tops harvesting unit which is frontally hung on the aggregating wheel integral tractor at a size of amplitude of fluctuations in the longitudinally vertical plane of the cutting device it is necessary to construct its mathematical model.

For this purpose we will analytically consider the movement of the beet tops harvesting unit only in the longitudinal-vertical plane, i.e. we will construct mathematical model of fluctuation of the beet tops harvesting unit at the movement on roughnesses of a surface of the soil only in one plane. On the basis of [1] we will make first of all the equivalent scheme of the movement frontally hung on the aggregating wheel integral tractor of the beet tops harvesting unit in the longitudinally vertical plane (fig. 2).

Apparently from fig.2, the beet tops harvesting unit joins the aggregating tractor by means of two lower drafts OK and one upper draft DM , having hinges in points O, D, M and K . We will designate radiuses of gage wheels and the cutting device respectively through R and R_1 . We will designate the mass of all beet tops harvesting unit through M ; the mass of two gage wheels — through $m = m_1 + m_2$ (where m_1 — mass of the first gage wheel, m_2 — масса second gage wheel). Weight m both gage wheels it is concentrated in a point B . Gravity of the beet tops harvesting unit which is enclosed in its center of masses (a point C) — as $\overline{G}$.

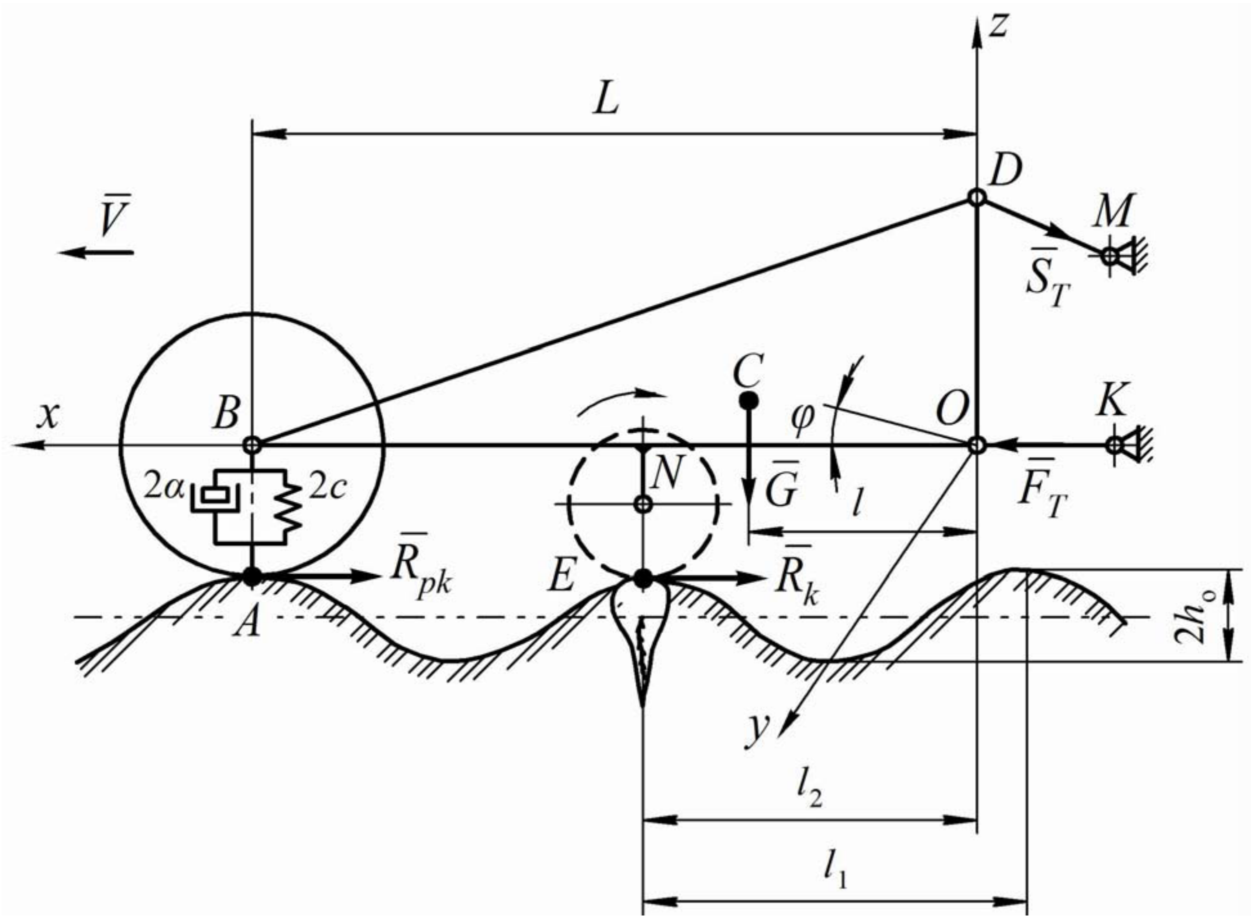

Fig. 2. — The equivalent scheme frontally hung on the aggregating integral tractor of the beet tops harvesting unit

Let's carry this dynamic system to motionless Cartesian coordinates $XOYZ$. Thus plane XOZ matches the longitudinal plane of the beet tops harvesting unit and is the vertical plane to a field surface.

matches the longitudinal plane of the beet tops harvesting unit and is the vertical plane to a field surface $2c$ and total coefficient of damping 2α .

We consider that gage wheels, in general, at the movement in row-spacings of crops of sugar beet rumple upper (more friable) layer of a surface of the soil, however move on roughnesses which have the longitudinal profile close to sinusoidal type.

As a first approximation we can consider that a pneumatic gage wheel, moving in row-spacings of crops of sugar beet and rumpling an upper loose coating of the soil, contacts to roughness of a surface of a field in a point $A.$ Thus roughnesses of a surface of the soil (in a smoothed look) can be presented in the form of harmonious function, i.e. analytical expression of such look [2]:

$$h = h_o\left(1 - \cos\frac{2\pi x}{l_1}\right), \qquad (1)$$

where h — ordinate of height of roughness of a surface of the soil, m, h_o — half of height of roughness of a surface of the soil, m, l_1 — half of height of roughness of a surface of the soil, m; $x = Vt$ – current coordinate, m; V – forward speed of the movement of the beet tops harvesting unit, m·s^{-1}.

Thanks to the weight, the tractor, at the movement on roughnesses of a surface of the soil, rumples the land wheels an upper layer of earth even more significantly, smoothing thereby the existing roughnesses that promotes reduction of amplitude of vertical fluctuations of the center of mass of the tractor.

It is obvious that for the specified reasons vertical fluctuations of the wheel aggregating tractor considerably weaken, however, it is necessary to believe, do not disappear at all. Therefore hinges in points K and M (fig. 2), as belonging to a tractor, also make vertical fluctuations. However thanks to availability of hinges in points O and D and the considerable mass of the beet tops harvesting unit, these fluctuations are practically not transferred to the beet tops harvesting unit. Fluctuations of the aggregating tractor cause only angular fluctuations of drafts OK and DM, attaching the beet tops harvesting unit to a tractor. Thus it is almost possible to consider that hinges O and D do not make vertical fluctuations, and draft OK and DM do not make vertical fluctuations, and draft O and D respectively, and other ends (hinges K and M) fluctuate independently together with a tractor. Owing to told above, we can consider that points of subweight of the beet tops harvesting unit (hinges O and D) move as a first approximation rectilinearly and evenly.

As the beet tops harvesting unit is hung ahead of the aggregating row-crop tractor, its gage wheels first of all perceive the existing roughnesses of a profile of a surface of a field that just, and causes vertical angular fluctuations of a frame of the beet tops harvesting unit round a point O. It is obvious that an angle of rotation of a frame of the beet tops harvesting unit round a point O significantly depends on size h roughnesses in a point A, in which there is a gage wheel at present. Therefore this corner φ in some approach we can define from such expression:

$$h = L \cdot \varphi, \qquad (2)$$

where L — distance from a point B axes of a gage wheel to a subweight point O frames to the lower draft OK (fig. 2).

From expression (2) we find value of this corner φ:

$$\varphi = \frac{h}{L}, \qquad (3)$$

or, considering expression (1), we receive final expression for a corner φ:

$$\varphi = \frac{h_o}{L}\left(1 - \cos\frac{2\pi x}{l_1}\right). \qquad (4)$$

Thus, roughnesses of a surface of a field are the kinematic activator of angular fluctuations of a frame of the beet tops harvesting unit. However, turn of a frame of the car round a point O can carry out only the moment of some force which line of action does not pass through a point O. Such force in this case is force generated by these roughnesses of a surface of the soil which is obvious, will change under the same sinusoidal law, as roughnesses of a profile of a surface of a field, i.e., according to such expression:

$$H = H_o\left(1 - \cos\frac{2\pi x}{l_1}\right), \qquad (5)$$

where H_o — amplitude of the specified force.

It is obvious that this force is applied in a point A, it is directed along an axis OZ, it is directed along an axis O it is directed along an axis

$$M_o(H) = H_o L\left(1-\cos\frac{2\pi x}{l_1}\right). \qquad (6)$$

This force is the external active force operating on a frame of the beet tops harvesting unit from a field surface.

Except this main force, in a point A horizontal reaction is enclosed $\overline{R}_{pk}$, which also is, generally speaking, the variable having bigger value when moving a gage wheel up a sinusoid of a look (1) and smaller — when moving a gage wheel down the specified sinusoid. Also in a point E which also is, generally speaking, the variable having bigger value when moving a gage wheel up a sinusoid of a look (1) and smaller — when moving a gage wheel down the specified sinusoid. Also in a point $\overline{R}_k$ resistance to cutting of a tops of vegetable rotor beet tops harvesting unit. It is obvious that these two forces are of little importance in creation of angular fluctuations of a frame of the car in comparison with force $\overline{H}$ and weight $\overline{G}$ the most beet tops harvesting unit. Forces $\overline{R}_{pk}$ and $\overline{R}_k$ first of all create resistance to movement of the beet tops harvesting unit across the field, and are sent to opposite driving force $\overline{F}_T$ from the aggregating tractor. Force $\overline{S}_T$ draft tension DM also does not play an essential role in creation of angular fluctuations of a frame of the beet tops harvesting unit owing to the trifle. Driving force $\overline{F}_T$ in general crosses a point O , and therefore only pushes the beet tops harvesting unit forward, without creating what moment of turn of a frame of the beet tops harvesting unit round a point O .

Thus, the essential role in creation of angular fluctuations of a frame of the beet tops harvesting unit in the vertical plane is played only by force $\overline{H}$ (kinematic excitement), arising owing to availability of roughnesses of a surface of a field, and force $\overline{G}$ weight of the beet tops harvesting unit. It should be noted that the line of action of these forces match the direction of movement of their points of application, that is points A and C respectively.

However it should be noted that amplitude H_o of force $\overline{H}$, and also reactions $\overline{R}_{pk}$ and $\overline{R}_k$ are unknown sizes. Therefore it is not possible to use the fundamental law of dynamics for drawing up the differential equations of the movement frontally hung on the wheel aggregating tractor of the beet tops harvesting unit taking into account roughnesses of a profile of a surface of a field. For the solution of this task it is reasonable to use the differential equations of the movement in the form of Lagrange of the II-nd sort [1].

For this purpose we will determine, first of all, the generalized coordinates of this dynamic system. Position of the center of mass of the beet tops harvesting unit (point) in the longitudinally vertical plane completely is defined by independent coordinate φ . As center of mass of pneumatic gage wheels (point B) carries out independent oscillating motions owing to it properties of gage wheels and ordinate of heights of roughness of a surface of the soil h considerably smaller, than length L , considerably smaller, than length Z . Thus, the considered oscillatory system can be given to two generalized coordinates:

$$q_1 = \varphi, \quad q_2 = Z. \qquad (7)$$

From told above follows that the main influence on vertical fluctuations of the beet tops harvesting unit show the elastoviscous resistance of tires of gage wheels, weight of the beet tops harvesting unit and the size of roughnesses of a surface of a field. Thus, there are all bases to consider that the studied mechanical system is affected by only potential forces and forces of viscous resistance. Let's use this circumstance by drawing up the differential equations of the movement of the considered dynamic system on the basis of the equations in the form of Lagrange of the II-nd sort.

It agrees [12] if the considered dynamic system is affected by only potential forces and forces of viscous resistance, the generalized forces Q_i, entering Lagrange's equation of the II-nd sort, are from such expression:

$$Q_i = -\frac{\partial P}{\partial q_i} - \frac{\partial R}{\partial \dot{q}_i}, \tag{8}$$

where P — potential energy of dynamic system; R — dissipative function (Rayleigh's function), q_i — the generalized coordinate, $\dot{q}_i$ — the generalized speed.

The differential equations of Lagrange of the II-nd sort in this case have the following appearance:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} = -\frac{\partial P}{\partial q_i} - \frac{\partial R}{\partial \dot{q}_i}, \tag{9}$$

where T — kinetic energy of this dynamic system.

The dynamic system considered in this work has two degrees of freedom and, therefore, as two generalized coordinates were stated above, $q_1 = \varphi$ and $q_2 = Z$. Therefore, as a result of it, we receive system of two differential equations in the form of Lagrange of the II-nd sort:

$$\left.\begin{aligned} \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\varphi}}\right) - \frac{\partial T}{\partial \varphi} &= -\frac{\partial P}{\partial \varphi} - \frac{\partial R}{\partial \dot{\varphi}}, \\ \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{Z}}\right) - \frac{\partial T}{\partial Z} &= -\frac{\partial P}{\partial Z} - \frac{\partial R}{\partial \dot{Z}}. \end{aligned}\right\} \tag{10}$$

Further, we will define components which log in the equations (10). So, kinetic energy T this mechanical system consists of kinetic energy of progress of the beet tops harvesting unit, angular movement of a frame of the car round a point O and kinetic energy of vertical fluctuations of its gage wheels. Therefore it will be equal:

$$T = \frac{MV^2}{2} + \frac{I_{oy}\dot{\varphi}^2}{2} + \frac{m\dot{Z}^2}{2}, \tag{11}$$

where M — mass of the beet tops harvesting unit, kg; V — speed of progress of the car, m/s; I_{oy} — the moment of inertia of the car concerning an axis OY, kg·m²; m – mass of gage wheels, kg.

Potential energy P this dynamic system it will be equal to work of elastic forces of deformation of pneumatic tires of both gage wheels and therefore is defined by the following expression:

$$P = c\left(L\varphi - Z\right)^2, \tag{12}$$

where c — coefficient of rigidity of pneumatic tires of wheels of the copying system, N/m; L — distance from an axis of subweight beet tops harvesting unit (a point O) to an axis of its gage wheels (point B), m.

Dissipative function R this dynamic system we define through forces of viscous resistance, proportional speeds of movement, and therefore the equal:

$$R = \alpha\left(L\dot{\varphi} - \dot{Z}\right)^2, \tag{13}$$

where α — coefficient of damping of gage wheels, N·s/m.

The specified forces of resistance are also caused by tires of gage wheels of the beet tops harvesting unit.

Let's execute the necessary transformations caused by application of the equations of dynamics in the form of Lagrange II-go sorts. For this purpose in the beginning we will find necessary derivatives which enter expression (10). For the first generalized coordinate φ they will be equal:

$$\frac{\partial T}{\partial \dot{\varphi}} = I_{oy}\dot{\varphi}, \tag{14}$$

Then

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\varphi}}\right) = I_{oy}\ddot{\varphi}. \tag{15}$$

If to consider that:

$$\frac{\partial T}{\partial \varphi} = 0, \tag{16}$$

then

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\varphi}}\right) - \frac{\partial T}{\partial \varphi} = I_{oy}\ddot{\varphi}. \tag{17}$$

Similarly:

$$\frac{\partial P}{\partial \varphi} = 2cL\left(L\varphi - Z\right), \tag{18}$$

and

$$\frac{\partial R}{\partial \dot{\varphi}} = 2\alpha L\left(L\dot{\varphi} - \dot{Z}\right). \tag{19}$$

For the second generalized coordinate Z let's find similar expressions. They will be equal:

$$\frac{\partial T}{\partial \dot{Z}} = m\dot{Z}, \tag{20}$$

then

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{Z}}\right) = m\ddot{Z}. \tag{21}$$

Further:

$$\frac{\partial T}{\partial Z} = 0, \tag{22}$$

and

$$\frac{\partial P}{\partial Z} = 2c\left(Z - L\varphi\right), \tag{23}$$

respectively:

$$\frac{\partial R}{\partial \dot{Z}} = -2\alpha\left(L\dot{\varphi} - \dot{Z}\right). \tag{24}$$

Let's substitute values of expressions (14) — (24) in system of the equations (10), we will receive:

$$\left.\begin{aligned} I_{oy}\ddot{\varphi} + 2cL\left(L\varphi - Z\right) + 2\alpha L\left(L\dot{\varphi} - \dot{Z}\right) = 0\,, \\ m\ddot{Z} + 2c\left(Z - L\varphi\right) - 2\alpha\left(L\dot{\varphi} - \dot{Z}\right) = 0\,. \end{aligned}\right\} \tag{25}$$

The received system (25) consisting of two differential equations models fluctuations of the center of mass of the beet tops harvesting unit (a point C) in the longitudinally vertical plane and fluctuations of the center of a gage wheel (point B).

Let's transform system (25) to such look:

$$\left.\begin{aligned} I_{oy}\ddot{\varphi} + 2cL^2\varphi - 2cLZ + 2\alpha L^2\dot{\varphi} - 2\alpha L\dot{Z} = 0\,, \\ m\ddot{Z} + 2cZ - 2cL\varphi - 2\alpha L\dot{\varphi} + 2\alpha\dot{Z} = 0\,, \end{aligned}\right\} \tag{26}$$

or it is final:

$$\left.\begin{aligned} \ddot{\varphi} + \frac{2cL^2}{I_{oy}}\varphi - \frac{2cL}{I_{oy}}Z + \frac{2\alpha L^2}{I_{oy}}\dot{\varphi} - \frac{2\alpha L}{I_{oy}}\dot{Z} = 0\,, \\ \ddot{Z} + \frac{2c}{m}Z - \frac{2cL}{m}\varphi - \frac{2\alpha L}{m}\dot{\varphi} + \frac{2\alpha}{m}\dot{Z} = 0\,. \end{aligned}\right\} \tag{27}$$

Thus, the system of the nonlinear differential equations (27) rather unknown generalized coordinates is received φ and Z , which represents settlement mathematical model of the movement frontally hung on the aggregating wheel integral tractor of the beet tops harvesting unit.

The system of the differential equations (27) with entry conditions can be solved according to the program made for this purpose on the personal computer by the adapted Runge-Kutt's method in MathCAD system.

However the received mathematical model is rather general. It is suitable for the description of vertical fluctuations of the beet tops harvesting unit which is frontally hung on a tractor in a case when the law of change of a profile of roughnesses of a surface of a field is unknown.

In this case the law of change of a profile of roughnesses of a surface of the soil is known and is set by expression (1). On the basis of this expression the expression (4) describing the law of change of angular coordinate is received above φ at forward movement of the beet tops harvesting unit. Therefore in this specific case expression (4) is the decision of system of the differential equations (27). Substituting this expression in one their equations of system (27), we receive the differential equation for definition of the law of change of coordinate Z .

Let's substitute expression (4) in the second differential equation of system (27). For this purpose we will write down expression (4) in the following look:

$$\varphi = \frac{h_o}{L}\left(1 - \cos\frac{2\pi Vt}{l_1}\right). \tag{28}$$

Let's substitute expression (4) in the second differential equation of system (27). For this purpose we will write down expression (4) in the following look t we receive:

$$\dot{\varphi} = \frac{2\pi h_o V}{L \cdot l_1} \cdot \sin\frac{2\pi Vt}{l_1}. \tag{29}$$

Substituting expressions (28) and (29) in the second equation of system (27), we receive the following differential equation of rather unknown function $Z\,(t)$:

$$\ddot{Z}+\frac{2\alpha}{m}\dot{Z}+\frac{2c}{m}Z=\frac{4\pi\alpha h_o V}{ml_1}\sin\frac{2\pi Vt}{l_1}-\frac{2ch_o}{m}\cos\frac{2\pi Vt}{l_1}+\frac{2ch_o}{m}. \qquad (30)$$

The equation (30) is the linear differential equation of the second order with constant coefficients with the right part.

Its common decision, as we know, consists of the common decision of the linear homogeneous equation and the private solution of the heterogeneous equation which look is defined by a type of the right member of equation (30):

$$Z=Z_{одн.}+Z^{*}. \qquad (31)$$

Let's find at first the common decision $Z_{одн.}$ homogeneous equation:

$$\ddot{Z}+\frac{2\alpha}{m}\dot{Z}+\frac{2c}{m}Z=0. \qquad (32)$$

The characteristic equation of the homogeneous equation (32) has such appearance:

$$k^{2}+\frac{2\alpha}{m}k+\frac{2c}{m}=0. \qquad (33)$$

Roots of this characteristic equation will accept the following value:

$$k_1=-\frac{\alpha}{m}+\sqrt{\frac{\alpha^{2}}{m^{2}}-\frac{2c}{m}},$$
$$k_2=-\frac{\alpha}{m}-\sqrt{\frac{\alpha^{2}}{m^{2}}-\frac{2c}{m}}. \qquad (34)$$

As in real dynamic system, i.e. in our specific case of value under roots in expressions have to be (34):

$$\frac{\alpha^{2}}{m^{2}}-\frac{2c}{m}<0,$$

that roots k_1 and k_2 буд ut complex numbers and therefore the common decision of the homogeneous differential equation (32) will have the following appearance:

$$Z_{одн.}=e^{-\frac{\alpha}{m}t}\left(C_1\sin\sqrt{\frac{\alpha^{2}}{m^{2}}-\frac{2c}{m}}\cdot t+C_2\cos\sqrt{\frac{\alpha^{2}}{m^{2}}-\frac{2c}{m}}\cdot t\right), \qquad (35)$$

where C_1 and C_2 – any constants.

Private decision Z^{*} the heterogeneous differential equation (30) we will look for in the following look:

$$Z^{*}=M\cdot\sin\frac{2\pi Vt}{l_1}+N\cdot\cos\frac{2\pi Vt}{l_1}+R, \qquad (36)$$

where M, N, R — unknown coefficients.

We find these coefficients method of uncertain coefficients. For this purpose we will carry out double differentiation of expression (36). We have:

$$\dot{Z}^{*}=M\frac{2\pi V}{l_1}\bullet\cos\frac{2\pi V}{l_1}t-N\frac{2\pi V}{l_1}\cdot\sin\frac{2\pi V}{l_1}t, \qquad (37)$$

$$\ddot{Z}^{*} = -M\frac{4\pi^2V^2}{l_1^2}\cdot\sin\frac{2\pi V}{l_1}t - N\frac{4\pi^2V^2}{l_1^2}\cdot\cos\frac{2\pi V}{l_1}t. \quad (38)$$

Substituting expressions (36), (37) and (38) in the differential equation (30), we will receive:

$$-M\frac{4\pi^2V^2}{l_1^2}\cdot\sin\frac{2\pi V}{l_1}t - N\frac{4\pi^2V^2}{l_1^2}\bullet\cos\frac{2\pi V}{l_1}t + \frac{2\alpha}{m}\left(M\frac{2\pi V}{l_1}\bullet\cos\frac{2\pi V}{l_1}t - N\frac{2\pi V}{l_1}\cdot\sin\frac{2\pi V}{l_1}t\right) + \frac{2c}{m}\left(M\cdot\sin\frac{2\pi Vt}{l_1} + N\cdot\cos\frac{2\pi Vt}{l_1} + R\right) =$$
$$= \frac{4\pi\alpha h_o V}{ml_1}\cdot\sin\frac{2\pi V}{l_1}t - \frac{2ch_o}{m}\cdot\cos\frac{2\pi V}{l_1}t + \frac{2ch_o}{m}. \quad (39)$$

In expression (39) we equate coefficients at the identical functions which are in the left and right part of the specified expression.

As a result we receive the following system of three algebraic linear equations concerning unknown M, N and R:

$$\begin{cases} \left(\frac{2c}{m} - \frac{4\pi^2V^2}{l_1^2}\right)\cdot M - \frac{4\pi\alpha V}{ml_1}\cdot N = \frac{4\pi\alpha h_o V}{ml_1}, \\ \frac{4\pi\alpha V}{ml_1}\bullet M + \left(\frac{2c}{m} - \frac{4\pi^2V^2}{l_1^2}\right)\cdot N = -\frac{2ch_o}{m}, \\ \frac{2c}{m}\cdot R = \frac{2c}{m}h_o. \end{cases} \quad (40)$$

From the last equation of system (40) it is found:

$$R = h_o. \quad (41)$$

Coefficients M and N we find from the first two equations of system (40), using Kramer's method.

We find the main determinant of system consisting of coefficients at unknown M and N two first equations of system (40). We have:

$$\Delta = \begin{vmatrix} \frac{2c}{m} - \frac{4\pi^2V^2}{l_1^2} & -\frac{4\pi\alpha V}{ml_1} \\ \frac{4\pi\alpha V}{ml_1} & \frac{2c}{m} - \frac{4\pi^2V^2}{l_1^2} \end{vmatrix} = \left(\frac{2c}{m} - \frac{4\pi^2V^2}{l_1^2}\right)^2 + \left(\frac{4\pi\alpha V}{ml_1}\right)^2. \quad (42)$$

We calculate further determinant Δ_M. It will be equal:

$$\Delta_M = \begin{vmatrix} \dfrac{4\pi\alpha h_o V}{m l_1} & -\dfrac{4\alpha\pi V}{m l_1} \\ -\dfrac{2ch_o}{m} & \dfrac{2c}{m} - \dfrac{4\pi^2 V^2}{l_1^2} \end{vmatrix} = -\frac{16\pi^3 \alpha V^3 h_o}{m l_1^3}. \qquad (43)$$

After that we find determinant Δ_N. After that we find determinant:

$$\Delta_N = \begin{vmatrix} \dfrac{2c}{m} - \dfrac{4\pi^2 V^2}{l_1^2} & \dfrac{4\pi\alpha h_o V}{m l_1} \\ \dfrac{4\alpha\pi V}{m l_1} & -\dfrac{2ch_o}{m} \end{vmatrix} = -\frac{2ch_o}{m}\left(\frac{2c}{m} - \frac{4\pi^2 V^2}{l_1^2}\right) - \frac{16\pi^2 \alpha^2 V^2 h_o}{m^2 l_1^2}. \qquad (44)$$

Using Kramer's rule, we find coefficients M and N:

$$M = \frac{\Delta_M}{\Delta} = -\frac{16\pi^3 \alpha V^3 h_o}{m l_1^3 \left[\left(\dfrac{2c}{m} - \dfrac{4\pi^2 V^2}{l_1^2}\right)^2 + \dfrac{16\pi^2 \alpha^2 V^2}{m^2 l_1^2}\right]}, \qquad (45)$$

$$N = \frac{\Delta_N}{\Delta} = -\frac{\dfrac{2ch_o}{m}\left(\dfrac{2c}{m} - \dfrac{4\pi^2 V^2}{l_1^2}\right) + \dfrac{16\pi^2 \alpha^2 V^2 h_o}{m^2 l_1^2}}{\left(\dfrac{2c}{m} - \dfrac{4\pi^2 V^2}{l_1^2}\right)^2 + \dfrac{16\pi^2 \alpha^2 V^2}{m^2 l_1^2}}. \qquad (46)$$

Let's substitute the received expressions (41), (45) and (46) in (36), we will receive the private decision Z^* heterogeneous differential equation (30). We have:

$$Z^* = -\frac{16\pi^3 \alpha V^3 h_o}{m l_1^3 \left[\left(\dfrac{2c}{m} - \dfrac{4\pi^2 V^2}{l_1^2}\right)^2 + \dfrac{16\pi^2 \alpha^2 V^2}{m^2 l_1^2}\right]} \cdot \sin \frac{2\pi V}{l_1} t - \frac{\dfrac{2ch_o}{m}\left(\dfrac{2c}{m} - \dfrac{4\pi^2 V^2}{l_1^2}\right) + \dfrac{16\pi^2 \alpha^2 V^2 h_o}{m^2 l_1^2}}{\left(\dfrac{2c}{m} - \dfrac{4\pi^2 V^2}{l_1^2}\right)^2 + \dfrac{16\pi^2 \alpha^2 V^2}{m^2 l_1^2}} \cdot \cos \frac{2\pi V}{l_1} t + h_o. \qquad (47)$$

Expression (47) defines the law of forced fluctuations of the center of mass of gage wheels of the beet tops harvesting unit.

Considering expressions (31), (35) and (36) we will write down the common decision of the differential equation (30):

$$Z = e^{-\frac{\alpha}{m}t}\left(C_1 \sin\sqrt{\frac{\alpha^2}{m^2} - \frac{2c}{m}}\cdot t + C_2 \cos\sqrt{\frac{\alpha^2}{m^2} - \frac{2c}{m}}\cdot t\right) + \\ +M\cdot\sin\frac{2\pi V}{l_1}t + N\cdot\cos\frac{2\pi V}{l_1}t + R\,, \tag{48}$$

where M, N and R — are defined from expressions (45), (46) and (41) respectively.

Any constants of integration C_1 and C_2 let's define from the following entry conditions:

$$\text{at } t = 0:\ z = 0\ ,\ \dot{z} = 0\,. \tag{49}$$

For this purpose we will differentiate on time t expression (48). We have:

$$\dot{Z} = -\frac{\alpha}{m}e^{-\frac{\alpha}{m}t}\left(C_1 \sin\sqrt{\frac{\alpha^2}{m^2} - \frac{2c}{m}}\cdot t + C_2 \cos\sqrt{\frac{\alpha^2}{m^2} - \frac{2c}{m}}\cdot t\right) + \\ +e^{-\frac{\alpha}{m}t}\cdot\left(C_1\sqrt{\frac{\alpha^2}{m^2} - \frac{2c}{m}}\bullet\cos\sqrt{\frac{\alpha^2}{m^2} - \frac{2c}{m}}\cdot t - C_2\sqrt{\frac{\alpha^2}{m^2} - \frac{2c}{m}}\cdot\sin\sqrt{\frac{\alpha^2}{m^2} - \frac{2c}{m}}\cdot t\right) + \\ +M\cdot\frac{2\pi V}{l_1}\cos\frac{2\pi V}{l_1}t - N\cdot\frac{2\pi V}{l_1}\sin\frac{2\pi V}{l_1}t\,. \tag{50}$$

Let's substitute values of entry conditions (49) in expressions (48) and (50), we receive the following system of the linear equations concerning unknown C_1 and C_2:

$$\left.\begin{aligned} & C_2 + N + R = 0, \\ & -\frac{\alpha}{m}C_2 + C_1\sqrt{\frac{\alpha^2}{m^2} - \frac{2c}{m}} + \frac{2\pi V}{l_1}M = 0. \end{aligned}\right\} \tag{51}$$

From system of the equations (51) we find:

$$C_1 = \frac{\frac{\alpha}{m}(N + R) + \frac{2\pi V}{l_1}M}{\sqrt{\frac{\alpha^2}{m^2} - \frac{2c}{m}}}, \qquad C_2 = -(N + R). \tag{52}$$

Substituting values of any constants (52) in expression (48), finally we find the common decision of the differential equation (30):

$$z = e^{-\frac{\alpha}{m}t}\left(\frac{\frac{\alpha}{m}(N+R)+\frac{2\pi V}{l_1}M}{\sqrt{\frac{\alpha^2}{m^2}-\frac{2c}{m}}}\cdot\sin\sqrt{\frac{\alpha^2}{m^2}-\frac{2c}{m}}\cdot t - (N+R)\cdot\cos\sqrt{\frac{\alpha^2}{m^2}-\frac{2c}{m}}\cdot t\right)+M\cdot\sin\frac{2\pi V}{l_1}t+N\cdot\cos\frac{2\pi V}{l_1}t+R, \tag{53}$$

where M , N and R — are defined from expressions (45), (46) and (41) respectively.

Expression (53) defines the law of forward vertical fluctuations of the center of mass of gage wheels (point) at the movement of the beet tops harvesting unit on the roughnesses of a surface of the soil set by analytical expression (1).

Apparently from expression (53), amplitude of own fluctuations of the center of mass of gage wheels will be equal:

$$A = \sqrt{\frac{\left[\frac{\alpha}{m}(N+R)+\frac{2\pi V}{l_1}M\right]^2}{\frac{\alpha^2}{m^2}-\frac{2c}{m}}+(N+R)^2}, \tag{54}$$

amplitude of forced fluctuations of the center of mass of gage wheels will be equal:

$$B = \sqrt{M^2+N^2}, \tag{55}$$

where M , N and R — are defined from expressions (45), (46) and (41) respectively.

Circular frequency ν of own fluctuations of the center of gage wheels is from such expression:

$$\nu = \sqrt{\frac{\alpha^2}{m^2}-\frac{2c}{m}}. \tag{56}$$

Circular frequency ω of forced fluctuations it will be equal:

$$\omega = \frac{2\pi V}{l_1}. \tag{57}$$

Having determined coordinate Z vertical movement of the center of mass of gage wheels, according to expression (53), we can define vertical movement of a knife of the rotor cutting device (a point E , fig. 2) from the following expression:

$$S_E(t) = l_2\varphi(t) - \frac{l_2}{L}Z(t), \tag{58}$$

where l_2 – distance from a point M fastenings of a knife to a frame to a point O the subweight of the beet tops harvesting unit to the lower draft OK ; $\varphi(t)$ — is defined, according to expression (28).

Vertical movement of the center of mass of the beet tops harvesting unit (point C) Vertical movement of the center of mass of the beet tops harvesting unit (point:

$$S_C(t) = l\varphi(t) - \frac{l}{L}Z(t), \tag{59}$$

where l — distance from the vertical axis passing through a point C , to point O ; $\varphi(t)$ — is defined according to expression (28).

Moreover, vertical movement $S(t)$ any point of the frame which is at distance x from a point O , from a point:

$$S(t) = x\varphi(t) - \frac{x}{L}Z(t). \qquad (60)$$

At numerical modeling on the personal computer of value of speed of the movement of a tractor with which the beet tops harvesting unit is aggregated, changed from V = 10 km/h to V = 12 km/h. Also different values of the moments of inertia of the beet tops harvesting unit were used I_{oy} (taking into account the mass of the tops of vegetable which is in the car) which depend from M — mass of the beet tops harvesting unit and L — distances from an axis of gage wheels to a subweight point O (fig. 2). At calculations some values of the moments of inertia got out: from $I_{oy} = 30\ \mathrm{kg} \cdot \mathrm{m}^2$ to $I_{oy} = 60\ \mathrm{kg} \cdot \mathrm{m}^2$.

Conclusions

1. The new machine and tractor unit in which the beet tops harvesting unit which is frontally installed on wheel row crop an integral tractor carries out a continuous non-sensing cut of the main array of a tops of vegetable from all width of its capture is developed.

2. The beet tops harvesting unit which is frontally installed on the aggregating tractor makes, in the course of work, angular fluctuations in the longitudinally vertical plane at the movement on roughnesses of a surface of the soil which are kinematic activators, and also independent oscillating motions owing to the uprugodempfiruyushchy properties of the pneumatic supporting wheels moving in row-spacings of crops of sugar beet and having the corresponding coefficients of rigidity and damping.

3. For the considered dynamic system the equivalent scheme of the movement in the longitudinally vertical plane with designation of all forces operating on it, the constructive sizes and the set system of coordinates, and also a choice of the generalized coordinates is constructed.

4. On the basis of the initial equations of dynamics in the form of Lagrange of the 2nd sort the system of two nonlinear differential equations of fluctuations frontally hung on the aggregating wheel integral tractor of the beet tops harvesting unit, rather unknown generalized coordinates φ and Z which represents settlement mathematical model of its movement is received.

5. The carried-out analytical transformations gave the chance to receive the common decision of the differential equation, to find the law of forward vertical fluctuations of the center of mass of gage wheels at the movement of the beet tops harvesting unit on roughnesses of a surface of the soil, to find final expressions of amplitude of own and forced fluctuations of the center of mass of supporting wheels, and also their circular frequency of own and forced fluctuations.

6. For further numerical modeling on the personal computer of design and kinematic data, frontally hung on the wheel aggregating row-crop tractor, beet tops harvesting unit the final analytical expression allowing to receive vertical movement of any point of its frame is received (including lower end of the rotor cutting device) at fluctuations of the beet tops harvesting unit in the course of its work.

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